Math 2X03 - Homework 4

Due: June 02, 2016

Chapters Covered: Chapter 16.1, 16.2, 14.6 - 14.8

1. The temperature T in a metal ball is inversely proportional to the distance from the center of the ball, which we take to be the origin. The termperature at the point (1,2,2) is 120°.
   (a) Find the rate of change of T at $(1, 2, 2)$ in the direction toward the point $(2, 1, 3)$.
   (b) Show that at any point in the ball the direction of greatest increase in temperature is given by a vector that points toward the origin.
2. Find the equation of the tangent plane and the normal line to the surface:
$$x^2 - 2y^2 + z^2 + yz = 2,$$
   at the point $(2, 1, -1)$.
3. Find the local maximum and local minimum values and saddle point(s) of the function.
   (a) $f(x, y) = (1 + xy)(x + y)$
   (b) $f(x, y) = x \sin y$
4. Find the absolute maximum and minimum values of f on the given set D.
   (a) $f(x, y) = 2x^3 + y^4$, $D = \{(x, y) \,|\, x^2 + y^2 \leq 1\}$.
   (b) $f(x, y) = 1 + 4x - 5y$, D is the closed triangular region with vertices $(0, 0)$, $(2, 0)$ and $(0, 3)$.
5. Find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in the ellipsoid
$$9x^2 + 36y^2 + 4z^2 = 36$$
6. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraints:
   (a) $f(x, y, z) = xyz$ subject to constraint $x^2 + 2y^2 + 3z^2 = 6$
   (b) $f(x, y, z) = yz + xy$ subject to the constraints $xy = 1$ and $y^2 + z^2 = 1$.
7. The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on the ellipse that are nearest to and farthest from the origin.
8. (Chapter 16.1 # 36) The **flow lines** (or **streamlines**) of a vector field are the paths followed by a particle whose velocity vector field is the given vector field. Thus, the vectors in the vector field are tangent to the flow-lines.
   (a) Use a sketch of the vector field $\mathbf{F}(x, y) = x\mathbf{i} - y\mathbf{j}$ to draw the some flow lines. (You can use computer program to draw the flow lines) From your sketches, can you guess the equation of the flow lines?

(b) If parametric equation of a flow line are $x = x(t)$, $y = y(t)$, explain why these functions satisfy the differential equations $\dfrac{dx}{dt} = x$ and $\dfrac{dy}{dt} = -y$. Then solve the differential equations to find an equation of the flow line that passes through the point $(1, 1)$

9. Evaluate the line integral, where C is the given curve

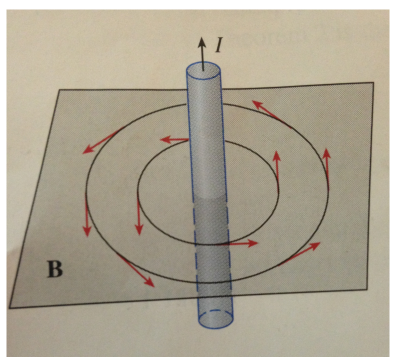
   (a) $\displaystyle\int_C xy^4\, ds$, where C is the right half of the circle $x^2 + y^2 = 16$

   (b) $\displaystyle\int_C (x+yz)\, dx + 2x\, dy + xyz\, dz$, where C consists of the line segments from $(1, 0, 1)$ to $(2, 3, 1)$ and from $(2, 3, 1)$ to $(2, 5, 2)$.

10. A thin wire is bent into the shape of a semicircle $x^2+y^2 = 4$, $x \geq 0$. If the linear density is constant k, find the mass and center of mass of the wire.

11. Find the work done by the force field $\mathbf{F}(x) = \langle y + z, x + z, x + y\rangle$ on a particle that moves alongs the line segment from $(1, 0, 0)$ to $(3, 4, 2)$.

12. (Chapter 16.2 # 52) Experiments show that a steady current I in long wire produces a magnetic field $\mathbf{B}$ that is tangent to any circle that lies in the plane perpendicular to the wire and whose center is the axis of the wire (as shown in the figure below).



Ampere's Law relates the electric current to its magnetic effects and states that

$$\int_C \mathbf{B} \cdot d\mathbf{r} = \mu_0 I$$

where I is the net current that passes through any surface bounded by a closed curve C and μ_0 is a constant called the permiability of free space. By taking C to be a circle with radius r, show that the magnitude $B = |\mathbf{B}|$ of the magnetic field at a distance r from the center of the wire is

$$B = \frac{\mu_0 I}{2\pi r}$$